The validity of maximum force predictions based on single-joint torque measurements

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Abstract

Sophisticated digital human models (DHMs) allow for strength prediction in arbitrary postures. In most cases the maximum force is calculated based on joint- and posture-specific joint torques. This approach is also used in multi-joint cases, while it has not been assessed so far if this procedure is justified or could lead to biased results. Differences between single- and multi-joint cases should not be dismissed without further investigations based on bi-articular muscles, multifunctional muscles, co-contractions, and inhibition effects.

This paper is dedicated to this open question and examines the differences between real maximum forces and maximum force predictions based on single-joint torque measurements. Therefore, 19 young males performed right arm pulling and pushing maximum force measurements in two different postures. Maximum elbow and shoulder joint torques were known for each posture and each subject. Consequently, the corresponding maximum forces could be predicted using an optimization study in AnyBody. A reconstruction of the multi-joint pulling and pushing trials in AnyBody helped to discuss arising differences in terms of muscle activations.

The results show that maximum forces were on average overestimated by approximately 20%. This new knowledge should be taken into account in strength models of DHMs.

Keywords: Joint torque, Strength, Digital human model, Biomechanics, Hand-arm-system, Physical capacity, Elbow, Shoulder, Force

1. Introduction

Common digital human models (DHMs) like RAMSIS, 3DSSPP, or Jack allow for maximum force prediction of end-effectors based on single-joint torque measurements. For example for the elbow a huge amount of studies can be found (Clarke et al. 1950; Doss and Karpovich 1965; Singh and Karpovich 1968; Williams and Stutzman 1959; Sato and Sakai 1968; Petrofsky and Phillips 1980; van Zuylen et al. 1988; Leedham and Dowling 1995; Tsunoda et al. 1993; Winters and Kleweno, 1993; Linnamo et al. 2006; d'Souza et al., 2011; Guenzkofer et al., 2011; Guenzkofer et al., 2012). Based on static or dynamic equilibrium equations, the posture-dependent maximum joint torques of each participating degree of freedom will yield the corresponding maximum force.

Strength prediction of DHMs is often used for multi-joint conditions in which more than one joint is involved. However, due to bi-articular muscles spanning affected joints, inhibition effects, and co-contractions, the question arises if this simple approach is justified and which errors have to be expected. Imagine the task of pulling a handle. In this case the m. biceps brachii is needed for flexing the elbow while the m. triceps brachii is activated due to shoulder extension. Based on the synchronous activation of agonists and antagonists, the muscles could impede each other. This could lead to decreasing joint torques in the multi-joint compared to the single-joint condition and an overestimation of maximum external forces.

A look in the literature reveals a considerable amount of multi-joint force measurements concentrated on obtaining strength data of tasks involving the human arm (e.g. Rühmann and Schmidtke, 1989; Wakula et al., 2009; Nijhof and Gabriel, 2006; Castro et al., 2012). However, no study including correlations between single-joint and multi-joint strength could be found which would be of utter importance for the joint-specific joint torque modelling approach.

A more detailed look at muscular analyses of multi-joint tasks reveals the task-specific behaviour of bi-articular muscles (van Ingen Schenau, 1989; Jacobs and van Ingen Schenau, 1992; van Ingen Schenau et al., 1992; Paul and van Ingen Schenau, 1994; Rozendal, 1994; van Ingen Schenau et al., 1994; van Bolhuis et al., 1998; Osu and Gomi, 1999). The overall conclusions of these studies are that mono-articular muscles are responsible for work as well as ensuring the desired motions while bi-articular muscles control the force direction by distributing torques over the joints. Consequently, differences between single-joint and multi-joint cases could be conceivable even if not explicitly studied in terms of force output.

Finally, a very recent study of Hahn et al. (2011) has investigated the difference of single-joint and multi-joint strength curves for knee and ankle joint. Especially concerning the ankle joint the results

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were significantly different. Instead of an ascending limb in the single-joint condition, they obtained a plateau followed by a descending limb with an even higher maximum in the multi-joint case. They hypothesize that differences in muscle function could be due to counter bearing effects, redistribution of force by bi-articular muscles, and intermuscular force transmissions.
To sum it up, current research shows that muscles behave differently in the multi-joint case than in the isolated single-joint case. However, apart from a lot of research concerning the role of bi-articular and mono-articular muscles no single study could be found comparing multi-joint force output with predictions based on single-joint torques.
In order to examine the hypothesis that maximum force predictions based on single-joint torque measurements are overestimated, multi-joint maximum force measurements of right arm planar pushing and pulling tasks were performed using 19 young male subjects.
The objective of this experiment was to compare real and predicted maximum end-effector forces of right arm planar pushing and pulling tasks. As individual maximum elbow joint torque models were already available from the same subjects (Guenzkofer et al., 2012), only shoulder flexion and extension joint torque measurements had to be performed for the necessary set of single-joint torques.

2. Materials and Methods

2.1. Subjects

Nineteen subjects volunteered to take part in this study. All subjects were already used for obtaining elbow joint torque data in Guenzkofer et al. (2012). All subjects had to fulfil specific requirements. On the one hand competitive athletes were rejected as the results should approximate average population strengths at the best. On the other hand subjects had to be healthy and free of orthopaedic and neurological disorders. All subjects gave their written consent after having been informed about the purpose of the study and its procedures.

Table 1: Mean and standard deviation of age, height and weight of participants in each group

n	19
Age	25.6 ± 2.4 years
Height	181.9 ± 6.3 cm
Weight	77.3 ± 8.3 kg

2.2. Apparatus

In this chapter the devices for measuring single-joint shoulder joint torque and multi-joint strength are introduced. In both cases the torque sensor read-out was conducted using LabVIEW 8.5 (National Instruments Corporation, Austin, USA).

2.2.1. Single-Joint Shoulder Measurements

Shoulder joint torques were measured in a sitting posture using a purpose-built measuring device (Guenzkofer et al., 2012). Several adjustment possibilities ensure an adaptation to various anthropometries. The whole device is connected to a torque sensor (burster präzisionsmesstechnik gmbh & co kg, Gernsbach, Germany) which directly grabs shoulder flexion and extension joint torques (see figure 1). Therefore, the sensor axis (red dotted line in figure 1) has to be aligned to the anatomical shoulder flexion axis.

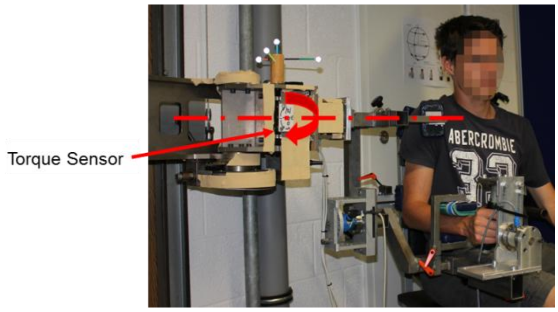


Figure 1: Shoulder joint torque measurement device

2.2.2. Multi-joint measurement

For the multi-joint force measurement subjects were seated and could be restrained by a full belt safety harness to a vertically adjusted backrest (figure 2). The test rig included height adjustable hand force dynamometers (Kistler Instrumente AG, Winterthur, Switzerland) which consist of three-component piezoelectric force sensors.

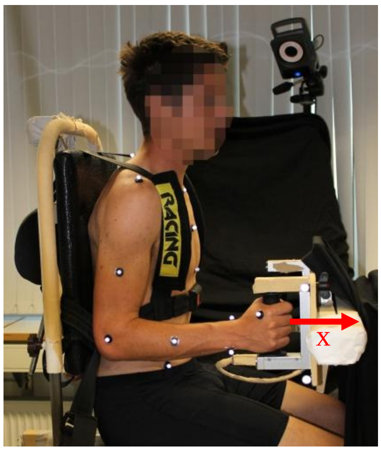
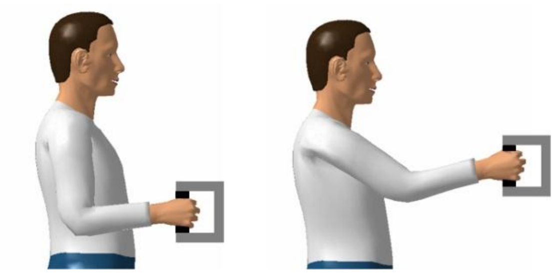


Figure 2: Multi-joint force measurement

2.3. Experimental Design

The measurement was performed in two different postures (figure 3). Posture 1 consists of zero degrees shoulder and 90 degrees elbow flexion and posture 2 of 60 degrees shoulder and 45 degrees elbow flexion.



Posture 1 Posture 2

Figure 3: Illustration of the two measurement postures

As only the individual maximum elbow joint torque equations of the subjects are known according to Guenzkofer et al. (2012), additional maximum shoulder joint torque measurements had to be performed within this study. Using the device explained in chapter 2.2.1, maximum shoulder flexion and extension joint torque measurements were conducted.

Isometric joint torque measurements were performed using the plateau-method, in which the subjects build up their maximum force in the first second and maintain the force for four seconds (Kumar 2004). The average value over a three seconds interval during the last four seconds matches the maximum value. In between two trials, the subjects had a rest period of two minutes. In order to ensure getting the real maximum torque value the subjects performed two trials in every position. Throughout the experiment the subjects did not receive any kind of motivation, neither verbal encouragement nor visual feedback (Mital and Kumar 1998; Brown and Weir 2001).

Eight maximum shoulder joint torque measurements per subject resulted from two torque directions, two trials per posture, and two postures.

Considering only torque vectors perpendicular to the sagittal plane, the influence of shoulder abduction, adduction, internal, and external rotation was neglected. As according to Al-Eisawi et al. (1994), wrist strength is never a limiting factor for whole-body exertions, maximum wrist joint torques were also neglected for maximum end-effector force prediction.

The multi-joint maximum strength tests were performed on the same day. Considering two postures, three trials per posture and two force directions resulted in 12 measurements in total. Again, the methods for isometric strength tests using the plateau method were used.

One measurement day consisted of two blocks: shoulder joint torque measurements and multi-joint force measurements. The order of the main blocks as well as the order of measurements within one block was completely randomized for all 19 subjects.

In order to examine muscular effects leading to potential differences between single-joint and multi-joint measurements, the multi-joint measurements were reconstructed using AnyBody (Damsgaard et al., 2006) by means of motion tracking using a six-camera Vicon MX T10 system (Oxford Metrics Ltd., Oxford, UK). Thereby, muscle activities of the multi-joint trials could be calculated and compared to the single-joint conditions.

2.4. Procedure

First, the participants were informed about the specific content of this experiment. Then, some anthropometric measures were taken and 16 retro-reflective markers were glued on specific anatomic landmarks. The shoulder measuring device as well as the multi-joint measuring device was adjusted to the specific anthropometry of the subject. Consequently, a warm-up phase using Thera-Bands or a dumbbell followed.

Then, based on the randomization the subjects either started with the isolated joint-specific shoulder torque measurements or the multi-joint force measurements. For the multi-joint isometric force measurements the same procedure as for the joint torque measurements was applied (cf. 2.3). For each force measurement, the subjects were instructed to perform the task only in the sagittal plane. For their better understanding they were instructed to pay attention that shoulder joint, elbow joint, and wrist joint lay on one plane. Additionally, they were instructed to exert forces exactly in the x-direction (figure 2).

2.5. Data Analysis

In this chapter the reconstruction from motion tracking and raw force data to calculated muscle activities using AnyBody is explained. Furthermore, the data processing for maximum force and muscle activity comparisons are presented.

2.5.1. Reconstruction using AnyBody

In a first step, raw motion tracking data are reconstructed and labelled in Vicon Nexus. The resulting labelled marker trajectories (x,y,z coordinates over time) are saved in a c3d-file. This c3d-file together with the anthropometric measurements and the csv-file containing the experimentally obtained forces is used as input for the AnyBody simulation. For the AnyBody simulation the GaitFullBody model from the public AnyBody Modelling Repository was used, which was modified by grounding the model at the pelvis and eliminating all body segments apart from trunk, head and right arm.

The AnyBody simulation consists of three parts. First, the individual anthropometric measurements serve for roughly scaling the models. Then, marker positions and body segment lengths are optimized

using a random rotatory calibration motion of each subject.
In the second step, the c3d-files of Vicon are used to reconstruct the real maximum force measurements considering the optimized anthropometry and marker positions from step 1.
Thirdly, joint torques, muscle forces, and muscle activities are calculated based on inverse dynamics. The required input consists of the obtained joint angles of step 2, the optimized anthropometry of step 1, and a csv-file containing the force data. The reaction force vector is attached to the glove point (middle point of the hand) of the AnyBody model. For the calculation of muscles forces, Hill-type muscles recruited by the MinMaxStrict criterion are used.

2.5.1. Comparison of Maximum Forces

In this section the main comparison between predicted and real maximum forces is described. Maximum external forces are predicted based on maximum shoulder and elbow joint torques using an optimization model implemented in AnyBody. In this case the variable of interest is the highest possible x-component of a maximum force in an arbitrary direction. This corresponds mathematically to a minimization of the inverted maximum force component in x-direction (1). The restrictions for the optimization are the lower and upper boundaries of maximum shoulder (2) and elbow (3) joint torques perpendicular to the sagittal plane.

Minimize $-F_{max}$ (1)

Subject to $T_{s,ext,max} < T_s < T_{s,flex,max}$ (2)

$T_{e,ext,max} < T_e < T_{e,flex,max}$ (3)

For the data processing only the highest maximum force of each condition was used. This results in one $F_{max,\ real}$ for each of the four situations for each subject.
For an accurate comparison of real and predicted forces the optimization model requires several inputs. First, the exact anthropometry and posture has to be used from the AnyBody reconstruction of the real multi-joint trials. Furthermore the exact posture was used for determining the individual maximum joint torques.
Using these inputs for the AnyBody optimization model led to maximum pulling and pushing force predictions for the two measurement postures.
The real and predicted maximum forces were analysed using a repeated-measures ANOVA using postures, force directions, and modes (real, predicted) as within-subject factors. The level of statistical significance was set at $p < .05$. All data were analyzed using the SPSS statistical package, release 18 (SPSS, Inc., Chicago, IL, USA).

2.5.1. Comparison of Muscle Activities

In the next step, possible coupling effects of simultaneous torque applications in adjacent joints should be examined. The elbow and shoulder joint torques calculated by AnyBody for the multi-joint trials serve as input for other AnyBody simulations in which exclusively shoulder or elbow joint torques are active. This means that due to the isolated consideration of exactly the elbow and shoulder joint torques obtained in the multi-joint trials, two isolated single-joint simulations had to be performed per multi-joint trial. Finally, differences in muscle activation for the same joint torques could explain differences between predicted and real external maximum forces.
Similarly to the comparison of maximum forces, exactly the same boundary conditions had to be used for the simulations. In this manner, the exact anthropometry, posture, and joint torques of the multi-joint AnyBody simulation served as input for the single-joint AnyBody simulation.
Finally, the multi-joint muscle activities can be compared to the single-joint ones.

3. Results

3.1. Comparison of Maximum Forces

Figure 4 depicts the comparison between predicted and experimentally obtained maximum external forces.

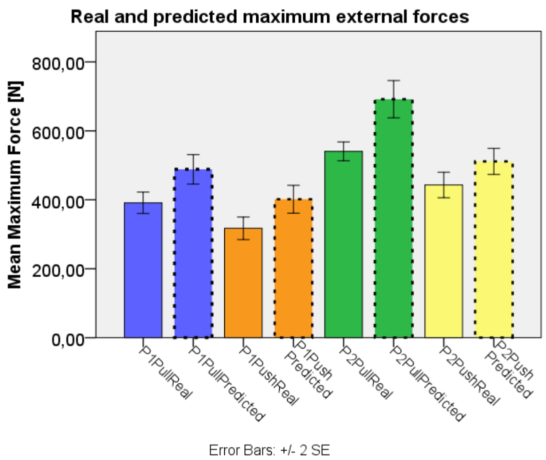


Figure 4: Diagram showing real and predicted maximum forces for two postures and two force directions. For example P2PushPredicted means: posture 2, push task, simulated result

A bar surrounded by a dotted line shows the maximum force prediction for the corresponding task. Pull forces generally significantly exceed push forces, $F(1, 18) = 83.64$, $p < .001$.
For all tasks, the maximum force predictions were significantly higher than the real measured forces, $F(1, 18) = 92.68$, $p < .001$. Moreover, in all cases forces are significantly higher in posture 2 than in posture 1, $F(1, 18) = 600.72$, $p < .001$.

In order to apply for DHMs, the differences between real and predicted maximum forces are presented as percentages in Table 2.

Table 2: Statistics of the force differences in [%]

	Min	Max	Mean	SD
P1Pull	-15	42	19	14
P1Push	-07	33	20	12
P2Pull	-13	33	20	12
P2Push	-43	49	11	21

All in all it seems that apart from pushing in posture 2, the predicted values are on average approximately 20% higher than the real measured values. However, it has to be considered that there is a wide deviation around each mean value. The negative signs of the minima show that for at least one subject in each task the real maximum forces were higher than the predicted ones.

3.2. Comparison of Muscle Activities

In this chapter the comparisons of muscle activities for the pulling task in posture 1 are presented (figure 5). For an easier understanding only selected elbow muscles are taken into account.

Blue bars without hatching represent muscle activities of shoulder and elbow joint torques acting simultaneously in the multi-joint condition. Green bars with single hatching show muscle activities that only have to produce the elbow torques obtained in the multi-joint trials. Lastly, yellow bars with cross hatch represent muscle activities of the single-joint shoulder torque simulation.

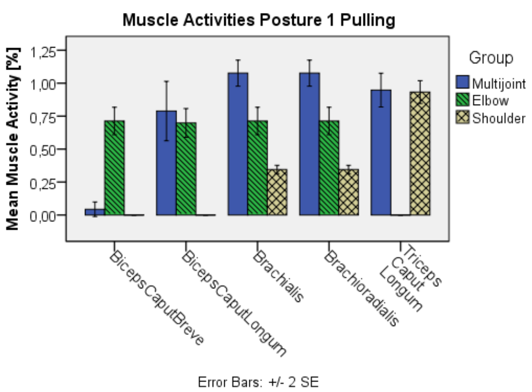


Figure 5: Muscle Activities for the same joint torques in the multi-joint and in the single-joint conditions of pulling in posture 1

Only producing an elbow flexion torque results in almost equal activations of m. biceps caput brevis, m. biceps caput longum, m. brachialis, and m. brachioradialis.

Exclusively producing shoulder extension torque of the multi-joint trials, only the long head of triceps is active from the group of the arm extensors. Interestingly, there is still a certain activation of m. brachialis and m. brachioradialis even if only shoulder extension is desired.

Looking at the blue bars without hatching reveals the situation in a multi-joint task in which shoulder extension as well as elbow flexion torques are needed. Obviously, the central nervous system or the muscle recruitment criterion, respectively, tries to reduce simultaneous agonist / antagonist activation. In comparison to only producing elbow joint torques, now activities are shifted from m. biceps caput breve to m. brachialis and m. brachioradialis. These muscles now mainly contribute to the desired joint torque while not impeding shoulder extension. The long head of the m. triceps is a significant shoulder extensor and cannot be restricted.

4. Discussion

The hypothesis was that due to biomechanical effects maximum force predictions based on maximum single joint torques will overestimate real maximum forces. Statistical tests unequivocally confirm this assumption.

To avoid too complex modelling and strength prediction, a planar task was chosen for this experiment. Thus, the subjects received a clear instruction, to keep their arms in the sagittal plane. Nevertheless, some subjects moved their elbow more laterally and consequently used another strategy for the task. A reason could be that they have only weak flexors, but strong rotators and abductors. Of course, they subconsciously chose a strategy that was more comfortable for them and led to a higher force output. This could be an explanation why for some subjects (table 2) the predicted maximum forces were below the real ones, as only the individual flexion/extension torques were restrictions in the optimization model.

The comparison of joint torques once acting separately and once combined with an adjacent joint torque revealed interesting insights in motor control.

A very interesting co-contraction occurred in the pure shoulder extension conditions. Even if only shoulder extension is desired - which is amongst others realized by an activation of the m. triceps caput longum - there is still a certain activation of the m. brachialis and m. brachioradialis. At first sight this may seem surprising as these are antagonists concerning the triceps. However, the triceps would extend the arm additionally to extending the shoulder. In order to compensate for the extending effect on the arm, co-contractions of the arm flexors m. brachialis and m. brachioradialis are needed. These muscles are consciously chosen as they do not impede shoulder extension as the bi-articular m. biceps brachii would do.

It was already explained in the results section that compared to the single-joint condition, there is an activity shift from m. biceps caput breve to m.

brachialis and m. brachioradialis in multi-joint pulling tasks. This is very clever as the m. biceps also flexes the shoulder which would handicap the desired shoulder extension by the m. triceps brachii. However, despite these compensation and adapted distribution strategies still a high simultaneous activation of m. biceps caput longum and m. triceps longum are present. This could be a main reason for the reduced maximum external pull forces compared to the predicted ones.
To sum it up, it seems that muscle recruitment tries to distribute muscle activities so that muscles impede each other the least. But nevertheless not all impediments can be avoided which results in lower external forces than predicted.

5. Conclusion

The conducted experiment succeeded in supporting the theory that due to biomechanical effects maximum force predictions based on joint torques obtained in isolated single-joint torques experiments overestimate realistic forces. Consequently, this new knowledge should be taken into account in strength models of DHMs.
For a posture consisting of zero degrees shoulder flexion and 90 degrees elbow flexion, a difference of approximately 20% could be detected. However, it has to be noted that this experiment only examined motions, in which extension in one joint is followed by flexion in the other joint and vice versa.
In tasks that require flexion or extension in both joints, no impeding effects are expected. For example when chopping wood the force generated by the triceps can be used for shoulder extension as well as for elbow extension. Thus, DHMs have to be aware of the kind of motion for a strength analysis or prediction.

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References

Al-Eisawi, K. W., Kerk, C. J., & Congleton, J. J. (1994). Limitations of Wrist Strength to Manual Exertion Capability in 2D Biomechanical Modeling. In Proceedings of the Human Factors and Ergonomics Society 38th Annual Meeting (pp. 595-563).

Brown, L. E.; Weir, J. P. (2001): ASEP Procedures recommendation 1: Accurate Assessment of muscular strength and power. In: Journal of Exercise Physiology (online) (Volume 4 Number 3).

Castro, C., La Vega, E. de, Báez, G., & Carrasco, F. (2012). Maximum force levels in different positions of shoulder and elbow. Work: A Journal of Prevention, Assessment and Rehabilitation 41, 41(Supplement 1/ 2012).

Clarke, H. H.; Elkins, E. C.; Martin, G. M.; Wakim, K. G. (1950): Application of muscle power to movements of the joints. In: Arch. Phys. Med. Rehabil. (31), S. 81–89.

Damsgaard, M., Rasmussen, J., Christensen, S. T., Surma, E., & Zee, M. de. (2006). Analysis of musculoskeletal systems in the AnyBody Modeling System. Simulation Modelling Practice and Theory, 14(8), 1100–1111. doi:10.1016/j.simpat.2006.09.001

Doss, W. S.; Karpovich, P. V. (1965): A comparison of concentric, eccentric, and isometric strength of elbow flexors. In: Journal of Applied Physiology 20 (2), S. 351–353.

d'Souza, S., Brückner, B., Rasmussen, J., & Schwirtz, A. (2011). Development of age and gender based strength scaled equations for use in simulation models. In Proceedings of the First International Symposium on Digital Human Modeling. Lyon, France.

Guenzkofer, F., Engstler, F., Bubb, H., & Bengler, K. (2011). Isometric elbow flexion and extension joint torque measurements considering biomechanical aspects. In Proceedings of the First International Symposium on Digital Human Modeling. Lyon, France.

Guenzkofer, F., Bubb, H., & Bengler, K. (2012). Maximum elbow joint torques for digital human models. Int. J. Human Factors Modelling and Simulation, Vol. 3, No. 2, pp.109–132.

Hahn, D., Olvermann, M., Richtberg, J., Seiberl, W., & Schwirtz, A. (2011). Knee and ankle joint torque-angle relationships of multi-joint leg extension. Journal of biomechanics, 44(11), 2059–2065. doi:10.1016/j.jbiomech.2011.05.011

Jacobs, R., & van Ingen Schenau, G. J. (1992). Control of an external force in leg extensions in humans. The Journal of physiology, 457, 611–626.

Kumar, S. (2004): Muscle strength. Boca Raton: CRC Press.

Leedham, J.S.; Dowling, J.J. (1995): Force-length, torque-angle and EMG-joint angle relationships of the human in vivo biceps brachii. In: Eur J Appl Physiol Occup Physiol 70 (5), S. 421–426.

Linnamo, V.; Strojnik, V.; Komi, P.V. (2006): Maximal force during eccentric and isometric actions at different elbow angles. In: Eur J Appl Physiol 96 (6), S. 672–678.

Mital, A.; Kumar, S. (1998): Human muscle strength definitions, measurement, and usage: Part I - Guidelines for the practitioner. In: International Journal of Industrial Ergonomics 22 (1-2), S. 101–121.

Nijhof, E. J., & Gabriel, D. A. (2006). Maximum isometric arm forces in the horizontal plane. Journal of Biomechanics, 39(4), 708–716.

Osu, R., & Gomi, H. (1999). Multijoint muscle regulation mechanisms examined by measured human arm stiffness and EMG signals. Journal of neurophysiology, 81(4), 1458–1468.

Paul, J. P., & van Ingen Schenau, G. J. (1994). Some muscles are more equal than others: An editorial introduction. Human Movement Science, 13(5), 539–542.

Petrofsky, J.S.; Phillips, C.A. (1980): The effect of elbow angle on the isometric strength and endurance of the elbow flexors in men and women. In: J Hum Ergol (Tokyo) 9 (2), S. 125–131.

Rozendal, R. H. (1994). Human poly-articular muscles: An anatomical comment. Human Movement Science, 13(5), 557–568. doi:10.1016/0167-9457(94)90005-1

Rühmann, H., & Schmidtke, H. (1989). Human strength: measurements of maximum isometric forces in industry. Ergonomics, 32(7), 865–879. doi:10.1080/00140138908966849

Sato, M.; Sakai, Y. (1968): Isometric and eccentric strength of elbow flexors and extensors. In: J. Anthropol. Soc. Nippon (76), S. 183–190.

Singh M; Karpovich PV (1968): Strength of forearm flexors and extensors in men and women. In: J Appl Physiol 25 (2), S. 177–180.

Tsunoda N; O'Hagan F; Sale DG; MacDougall JD (1993): Elbow flexion strength curves in untrained men and women and male bodybuilders. In: Eur J Appl Physiol Occup Physiol 66 (3), S. 235–239.

van Bolhuis, B. M., Gielen, C. C., & van Ingen Schenau, G. J. (1998). Activation patterns of mono- and bi-articular arm muscles as a function of force and movement direction of the wrist in humans. The Journal of physiology, 508 (Pt 1), 313–324.

van Ingen Schenau, G. J. (1989). From rotation to translation: Constraints on multi-joint movements and the unique action of bi-articular muscles. Human Movement Science, Vol. 8(4), 301–337.

van Ingen Schenau, G. J., Boots, P. J., Groot, G. de, Snackers, R. J., & van Woensel, W. W. (1992). The constrained control of force and position in multi-joint movements. Neuroscience, 46(1), 197–207.

van Ingen Schenau, G. J., Pratt, C. A., & Macpherson, J. M. (1994). Differential use and control of mono- and biarticular muscles. Human Movement Science, 13(3–4), 495–517. doi:10.1016/0167-9457(94)90051-5

van Zuylen, E.J.; van Velzen, A.; Denier van der Gon JJ (1988): A biomechanical model for flexion torques of human arm muscles as a function of elbow angle. In: J Biomech 21 (3), S. 183–190.

Wakula, J., Berg, K., Schaub, K., Bruder, R., Glitsch, U., & Ellegast, R. Isometric maximum forces of the whole-body in not upright postures and hand-finger-forces for the assembly-specific atlas. In 17th World Congress on Ergonomics (IEA 2009)

Williams, M.; Stutzmann, L. (1959): Strength variation through the range of motion. In: Phys. Ther. (39), S. 145–152.

Winters, J.M.; Kleweno, D.G. (1993): Effect of initial upper-limb alignment on muscle contributions to isometric strength curves. In: J Biomech 26 (2), S. 143–153.